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Resummation of angular dependent corrections in spontaneously broken gauge theories

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Abstract. Recent investigations of electroweak radiative corrections have revealed the importance of higher order contributions in high energy processes, where the size of typical corrections can exceed those associated with QCD considerably. Beyond one loop, only universal (angular independent) corrections are known to all orders except for massless $e^+e^- \longrightarrow f\overline{f}$ processes, where also angle dependent corrections exist in the literature. In this paper we present general arguments for the consistent resummation of angle dependent subleading (SL) logarithmic corrections to all orders in the regime where all invariants are still large compared to the gauge boson masses. We discuss soft isospin correlations, fermion mass and gauge boson mass gap effects, the longitudinal and Higgs boson sector as well as mixing contributions including CKM effects for massive quarks. Two loop results for the right handed Standard Model are generalized in the context of the high energy effective theory based on the standard model Lagrangian in the symmetric basis with the appropriate matching conditions to include the soft QED regime. The result is expressed in exponentiated operator form in a CKM extended isospin space in the symmetric basis. Thus, a full electroweak SL treatment based on the infrared evolution equation method is formulated for arbitrary high energy processes at future colliders. Comparisons with known results are presented.

1 Introduction

Future colliders in the TeV energy regime will attempt to clarify the physics responsible for electroweak symmetry breaking. In this context it is important to understand the radiative corrections from QCD as well as from the electroweak standard model (SM) sufficiently in order to disentangle new physics effects. For precision measurements in the percentile regime, higher order *electroweak* radiative corrections (two loops and possibly three) are crucial for this purpose [1–3].

The exponentiation of electroweak double logarithms (DL) [1] has now been established by independent two loop calculations in the fermionic sector [4–6] and therefore, using group theoretical arguments and the equivalence theorem, also in the longitudinal sector as was first derived in [7] via the infrared evolution equation method.

For universal, i.e. process independent subleading (SL) logarithms, the identity of the splitting function approach and the physical fields calculation was shown in [7,8] at the one loop level. In [9] general formulae were presented for arbitrary DL and SL corrections at one loop including process dependent angular terms. The importance of these latter corrections was also discussed at one loop in [10]. At higher orders, these corrections are important and they were first given for massless four fermion processes in [11]. For this particular group of processes even sub-subleading corrections have been calculated by employing QCD results and subtracting the QED corrections from the SM [12]. In both cases the effects of higher order subleading terms are large and need to be included in a consistent treatment.

In this paper, we are interested in calculating the angle dependent corrections to SL accuracy to all orders for arbitrary electroweak processes. These include fermion mass terms and the associated CKM mixing effects (which are absent for massless quarks since they are automatically mass eigenstates), other mixing and mass gap effects of the electroweak gauge bosons and the longitudinal sector. We assume that all invariants $2p_ip_j \gg M^2$, where M denotes the gauge boson mass with $M_Z \approx M_W \equiv M$.

Although we cannot directly use the framework of QCD for the SM, at high energies we can use a description based on the symmetric basis in which all terms with a mass dimension are neglected [2]. This formulation implies for instance in the neutral scalar sector that the relevant fields are ϕ_0 and $\phi_0{}^*$. While strictly speaking we are calculating corrections to amplitudes with these fields, the translation to the mass eigenstates is for the most part straightforward. In our example we have the relations

$$H(x) = \frac{1}{\sqrt{2}} \left(\phi_0{}^*(x) + \phi_0(x) \right),$$
$$\chi(x) = \frac{\mathrm{i}}{\sqrt{2}} \left(\phi_0{}^*(x) - \phi_0(x) \right). \qquad (1)$$

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Thus, the corrections factorize analogously for the mass eigenstates. Only in the neutral transverse sector we cannot directly assign a well-defined isospin. In this case, and for corrections involving gauge bosons only, we have to consider only the amplitude involving the non-Abelian field W^3 since the mixing contribution with the B field does not possess self-interactions.

The contributions from the regime below the scale M are due only to QED and can be incorporated via the appropriate matching conditions in the framework of the infrared evolution equation method [13].

The general approach we follow in this paper is the investigation of new effects introduced by spontaneous symmetry breaking in order to see if they can lead to novel angular contributions compared to the case of massless unbroken gauge theories. This is non-trivial and necessary in order to investigate the full electroweak SM. We use the term "broken gauge theories" in the sense that the local symmetry is hidden due to the degeneracy of the vacuum ground state and thus not evident in the physical states. The associated local BRST relations, however, still hold.

In Sect. 2 we give general two loop arguments leading up to a factorization in operator form for the angle dependent corrections in the physical basis. In Sect. 3 the results of Sect. 2 are generalized in the framework of the infrared evolution equation method based on the Lagrangian in the symmetric basis and matching at the weak scale. In Sect. 4, the results in the effective description are compared at one loop with results in the physical basis in order to clarify the method, and to the massless limit for four fermion processes also at higher orders. We summarize our results in Sect. 5.

2 General two loop arguments

Our general strategy in the following is to investigate if spontaneous symmetry breaking effects can lead to novel angle dependent SL corrections in comparison to the known contributions in massless unbroken gauge theories at two loop order. As discussed below, the results from the latter are used as input in our factorization for the electroweak angle dependent corrections.

In the SM, the novel effects arising due to the electroweak symmetry breaking mechanism are the mixing of the mass eigenstates, the existence of a scalar sector (which is not mass suppressed at high energies) and the mass gap of the electroweak gauge bosons. In addition to a massless non-Abelian theory there are massive particles and in practice, these mass terms regularize collinear divergences and are important for phenomenological predictions for future colliders.

The effect of mixing is important for photon and Z-boson final states, since an external Z-boson at one loop mixes with the photon field for instance. However, these mixing terms are not related to angle dependent corrections and are discussed in more detail in [9,7]. A new complication is in principle given by the CKM matrix elements, which enter in the couplings of massive quarks to the charged gauge bosons[1]. For universal corrections, using the conservation of the non-Abelian group charge, these terms are of the form

$$\sum_{k=1}^{3} V_{ik}^{*} V_{kj} f(m_i, m_k, m_j), \tag{2}$$

where $f(0,0,0) = 1$. For the exchange of the heavy gauge bosons, the mass terms only lead to additional mass suppressed contributions if $m_l \leq M$. The unitarity of the CKM matrix implies that the mass independent term in f leads to the unit matrix. For the general case with angle dependent corrections fermion mass terms remain, including in principle independent CKM matrix elements, and thus, we have to convince ourselves that at the n loop level to SL accuracy no new

$$\log^{2n-1} \frac{s}{m_j^2} \log \frac{s}{t}$$

arise from the CKM terms, where t denotes an invariant depending on the angle between the incoming and the outgoing particle for instance. These terms are discussed below but it should be noted that a massless calculation would not serve as a check of these corrections since in this case they are automatically mass eigenstates. For other final states, the mixing effects above the scale M correspond to a change of basis, namely the symmetric basis, and therefore do not give rise to new effects for physical cross sections.

The new ingredient in spontaneously broken gauge theories are the mass terms. In the case of the SM, the corrections from below the weak scale are only due to QED and the angular SL terms in particular are only due to photon exchange. Thus, at the two loop level, we have to consider corrections of the type depicted in Fig. 1 and show that the mass terms do not lead to new effects compared to those in the massless case. We limit ourseleves here to the situation in QED and the right handed SM.

The two scalar integrals of Fig. 1, regularized with gauge boson mass terms λ_1 and λ_2, are given in massive QED or in the case of right handed (massive) fermions by

$$\begin{aligned}
\mathrm{II}_\theta^a &= 4st \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \\
&\times \left\{ \frac{1}{(l_1^2 - \lambda_1^2)((p_1 - l_1)^2 - m_1^2)((p_1 - l_1 - l_2)^2 - m_1^2)} \right. \\
&\times \left. \frac{1}{((p_2 + l_1)^2 - m_2^2)(l_2^2 - \lambda_2^2)((p_3 + l_2)^2 - m_3^2)} \right\},
\end{aligned} \tag{3}$$

$$\begin{aligned}
\mathrm{II}_\theta^b &= 4st \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \\
&\times \left\{ \frac{1}{(l_1^2 - \lambda_1^2)((p_1 - l_2)^2 - m_1^2)((p_1 - l_1 - l_2)^2 - m_1^2)} \right. \\
&\times \left. \frac{1}{((p_2 + l_1)^2 - m_2^2)(l_2^2 - \lambda_2^2)((p_3 + l_2)^2 - m_3^2)} \right\},
\end{aligned} \tag{4}$$

[1] CKM matrix elements also occur in the scalar–quark–antiquark coupling. These diagrams, however, do not give rise to SL angular terms

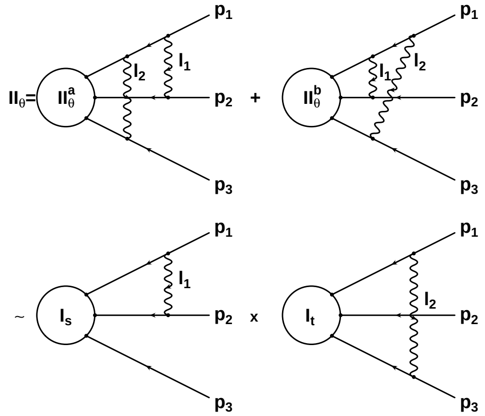

Fig. 1. Angular dependent two loop on-shell QED-like diagrams. The sum of II_θ^a and II_θ^b factorizes in massive QED or for the right handed SM into the product of the two one loop corrections (each with a different invariant and mass terms) in leading order

denoting $s = 2p_1p_2$, $t = 2p_1p_3$ and where the m_i are the masses of the external charged particles on their mass shell. Thus, it is straightforward to see that the sum of the two diagrams factorizes to leading order:

$$\begin{aligned}
\mathrm{II}_\theta^a + \mathrm{II}_\theta^b &= 4st \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \\
&\times \left\{ \frac{l_1^2 + l_2^2 - 2p_1(l_1+l_2)}{(l_1^2-\lambda_1^2)((p_1-l_1)^2-m_1^2)((p_1-l_1-l_2)^2-m_1^2)} \right. \\
&\times \frac{1}{((p_2+l_1)^2-m_2^2)(l_2^2-\lambda_2^2)} \\
&\times \left. \frac{1}{((p_1-l_2)^2-m_1^2)((p_3+l_2)^2-m_3^2)} \right\}, \\
&\approx \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \\
&\times \frac{2s}{(l_1^2-\lambda_1^2)((p_1-l_1)^2-m_1^2)((p_2+l_1)^2-m_2^2)} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \\
&\times \frac{2t}{(l_2^2-\lambda_2^2)((p_1-l_2)^2-m_1^2)((p_3+l_2)^2-m_3^2)}. \qquad (5)
\end{aligned}$$

The omitted cross term $2l_1l_2$ leads only to corrections containing three logarithms at the two loop level. It is thus on the same level as the approximation in the beginning of our discussion which only considers scalar integrals and can therefore be neglected. To DL accuracy we can employ the Sudakov technique, parametrizing the loop momenta along the external four momenta as

$$l_1 \equiv v_1 \left(p_1 - \frac{m_1^2}{s} p_2 \right) + u_1 \left(p_2 - \frac{m_2^2}{s} p_1 \right) + l_{1\perp}, \quad (6)$$

$$l_2 \equiv v_2 \left(p_1 - \frac{m_1^2}{t} p_3 \right) + u_2 \left(p_3 - \frac{m_3^2}{t} p_1 \right) + l_{2\perp}. \quad (7)$$

Thus, after rewriting the measure and integrating over the perpendicular components we find for the case of two photons with mass λ (omitting the principle value parts):

$$\begin{aligned}
\mathrm{II}_\theta^a + \mathrm{II}_\theta^b &\sim \frac{1}{8\pi^2} \left[\int_0^1 \frac{\mathrm{d}v_1}{v_1} \int_0^1 \frac{\mathrm{d}u_1}{u_1} \right. \\
&\times \left. \theta\left(su_1v_1 - \lambda^2\right) \theta\left(u_1 - \frac{m_1^2}{s} v_1\right) \theta\left(v_1 - \frac{m_2^2}{s} u_1\right) \right] \\
&\times \frac{1}{8\pi^2} \left[\int_0^1 \frac{\mathrm{d}v_2}{v_2} \int_0^1 \frac{\mathrm{d}u_2}{u_2} \right. \\
&\times \left. \theta\left(tu_2v_2 - \lambda^2\right) \theta\left(u_2 - \frac{m_1^2}{t} v_2\right) \theta\left(v_2 - \frac{m_3^2}{t} u_2\right) \right] \\
&= \frac{1}{8\pi^2} \left[\int_{\frac{\lambda^2}{s}}^1 \frac{\mathrm{d}v_1}{v_1} \int_{\frac{\lambda^2}{sv_1}}^1 \frac{\mathrm{d}u_1}{u_1} - \int_{\frac{\lambda^2}{s}}^{\frac{\lambda m_2}{s}} \frac{\mathrm{d}v_1}{v_1} \int_{\frac{\lambda^2}{sv_1}}^1 \frac{\mathrm{d}u_1}{u_1} \right. \\
&- \int_{\frac{\lambda m_2}{s}}^{\frac{m_2^2}{s}} \frac{\mathrm{d}v_1}{v_1} \int_{\frac{sv_1}{m_2^2}}^1 \frac{\mathrm{d}u_1}{u_1} - \int_{\frac{\lambda^2}{s}}^{\frac{\lambda m_1}{s}} \frac{\mathrm{d}u_1}{u_1} \int_{\frac{\lambda^2}{su_1}}^1 \frac{\mathrm{d}v_1}{v_1} \\
&- \left. \int_{\frac{\lambda m_1}{s}}^{\frac{m_1^2}{s}} \frac{\mathrm{d}u_1}{u_1} \int_{\frac{su_1}{m_1^2}}^1 \frac{\mathrm{d}v_1}{v_1} \right] \\
&\times \frac{1}{8\pi^2} \left[\int_{\frac{\lambda^2}{t}}^1 \frac{\mathrm{d}v_2}{v_2} \int_{\frac{\lambda^2}{tv_2}}^1 \frac{\mathrm{d}u_2}{u_2} - \int_{\frac{\lambda^2}{t}}^{\frac{\lambda m_3}{t}} \frac{\mathrm{d}v_2}{v_2} \int_{\frac{\lambda^2}{tv_2}}^1 \frac{\mathrm{d}u_2}{u_2} \right. \\
&- \int_{\frac{\lambda m_3}{t}}^{\frac{m_3^2}{t}} \frac{\mathrm{d}v_2}{v_2} \int_{\frac{tv_2}{m_3^2}}^1 \frac{\mathrm{d}u_2}{u_2} - \int_{\frac{\lambda^2}{t}}^{\frac{\lambda m_1}{t}} \frac{\mathrm{d}u_2}{u_2} \int_{\frac{\lambda^2}{tu_2}}^1 \frac{\mathrm{d}v_2}{v_2} \\
&- \left. \int_{\frac{\lambda m_1}{t}}^{\frac{m_1^2}{t}} \frac{\mathrm{d}u_2}{u_2} \int_{\frac{tu_2}{m_1^2}}^1 \frac{\mathrm{d}v_2}{v_2} \right] \\
&= \frac{1}{8\pi^2} \left[-\frac{1}{4} \log^2 \frac{s}{m_1^2} - \frac{1}{4} \log^2 \frac{s}{m_2} \right. \\
&+ \left. \frac{1}{2} \log \frac{s}{m_1^2} \log \frac{s}{\lambda^2} + \frac{1}{2} \log \frac{s}{m_2^2} \log \frac{s}{\lambda^2} \right] \\
&\times \frac{1}{8\pi^2} \left[-\frac{1}{4} \log^2 \frac{t}{m_1^2} - \frac{1}{4} \log^2 \frac{t}{m_3} \right. \\
&+ \left. \frac{1}{2} \log \frac{t}{m_1^2} \log \frac{t}{\lambda^2} + \frac{1}{2} \log \frac{t}{m_3^2} \log \frac{t}{\lambda^2} \right]. \qquad (8)
\end{aligned}$$

For the special case of $m_1 = m_2 = m_3$, (8) agrees with the two loop expanded expression of [14], where the all orders resummation of DL and SL terms are derived in e^-e^- scattering. The important point about the result in (8) is not only the factorized form in terms of the two massive one loop form factors but also the fact that the fermion mass terms correspond to each external on-shell line in the amplitude. Thus, by rewriting the term in the bracket of the last two lines in (8) as

$$\begin{aligned}
&-\frac{1}{4} \log^2 \frac{t}{m_1^2} - \frac{1}{4} \log^2 \frac{t}{m_3} + \frac{1}{2} \log \frac{t}{m_1^2} \log \frac{t}{\lambda^2} \frac{1}{2} \log \frac{t}{m_3^2} \log \frac{t}{\lambda^2} \\
&= \frac{1}{2} \log^2 \frac{t}{\lambda^2} - \frac{1}{4} \left(\log^2 \frac{m_1^2}{\lambda^2} + \log^2 \frac{m_3^2}{\lambda^2} \right) \qquad (9) \\
&\approx \frac{1}{2} \log^2 \frac{s}{\lambda^2} + \log \frac{s}{\lambda^2} \log \frac{t}{s} - \frac{1}{4} \left(\log^2 \frac{m_1^2}{\lambda^2} + \log^2 \frac{m_3^2}{\lambda^2} \right),
\end{aligned}$$

we see that the SL angular terms are indeed independent of the fermion mass terms. For the corrections involving the invariant $u \equiv 2p_2p_3$ the situation is analogous.

In the case of the SM with right handed massive fermions we need to consider in addition the exchange of Z-bosons. The results read

$$\begin{aligned} \mathrm{II}_\theta^a + \mathrm{II}_\theta^b \sim \frac{1}{8\pi^2} & \left[\frac{1}{2} \log^2 \frac{s}{M^2} \right] \\ \times \frac{1}{8\pi^2} & \left[\frac{1}{2} \log^2 \frac{s}{\lambda^2} + \log \frac{s}{\lambda^2} \log \frac{t}{s} \right. \\ & \left. - \frac{1}{4} \left(\log^2 \frac{m_1^2}{\lambda^2} + \log^2 \frac{m_3^2}{\lambda^2} \right) \right], \end{aligned} \quad (10)$$

for the case of a photon and a Z-boson with mass M, and for the case of two Z's we have

$$\begin{aligned} \mathrm{II}_\theta^a + \mathrm{II}_\theta^b \sim \frac{1}{8\pi^2} & \left[\frac{1}{2} \log^2 \frac{s}{M^2} \right] \\ \times \frac{1}{8\pi^2} & \left[\frac{1}{2} \log^2 \frac{s}{M^2} + \log \frac{s}{M^2} \log \frac{t}{s} \right]. \end{aligned} \quad (11)$$

Again we see the independence of the SL angular terms on the fermion mass terms and in addition, the fact that the gauge boson mass gap does not spoil the type of factorization in the right handed SM.

This type of factorization can be generalized on theoretical grounds to the situation in the full SM. It should be noted that all fermion mass singularities in the SM only arise through photon radiation or coupling renormalization. The latter is not important in our discussion here and is anyhow sub-subleading at higher orders. The exchange of the heavy gauge bosons does not lead to fermion mass singular terms assuming that all $m_i \leq M$ as can be explicitly seen in (10) and (11).

In order to generalize the Abelian type factorization derived in (8), (10) and (11) at the two loop level for diagrams involving fermion mass singularities, one has to consider in particular corrections such as the ones depicted in Fig. 2. Compared to QCD, these are novel type diagrams involving mixing effects in the fermionic sector through CKM matrix elements. The required factorized form shown there for the virtual diagrams[2], is actually a consequence of the type of fermion mass singular terms which occur in the associated real emission diagrams shown in the lower half of Fig. 2. The diagrams II_r^a and II_r^b do not produce fermion mass singular terms due to the virtuality ($\geq M$) of the line emitting the photon (wavy line). Thus, only the real emission diagrams in the last row remain which are of the required factorized form.

The connection between the real and virtual diagrams is now provided by the KLN theorem [15,16]. It states that, as a consequence of unitarity, transition probabilities are free of mass singularities when summed over all degenerate final states. This is true order by order in perturbation theory in renormalization schemes which do not introduce mass singular terms via the renormalization constants. As mentioned above, we are not concerned about the last point here.

Thus, the only way to cancel the real emission fermion mass singularities in Fig. 2 in fully inclusive cross sections is by the factorization of the associated virtual ones as

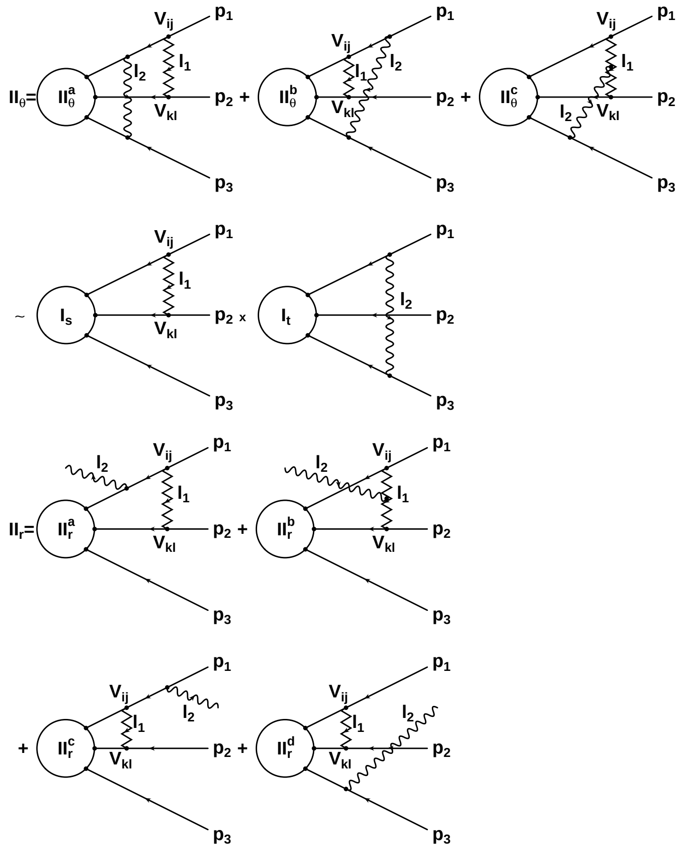

Fig. 2. Electroweak angle dependent two loop on-shell diagrams involving CKM matrix elements (denoted by V_{ij}). The wavy line denotes a photon, the zigzag line a $W^\pm$. The sum of the virtual diagrams II_θ^a, II_θ^b and II_θ^c (plus the relevant contributions from diagrams with gauge bosons exchanged between only two external legs omitted here for brevity) must factorize for the leading mass singular terms into the product of the two one loop corrections (each with a different invariant and mass terms). The formal reason for this factorization is provided by the fact that the real emission diagrams II_r^a and II_r^b are free of fermion mass singular terms due to the off-shellness generated by the $W^\pm$ exchange. Thus only the factorized real corrections of diagrams II_r^c and II_r^d contribute fermion mass singular terms and the KLN theorem then yields the analogous factorization for the above virtual corrections. It should be noted, however, that the diagram II_θ^c actually contributes not only to the ones depicted here, but obviously also to the ladder diagrams in which the photon couples to the external line labeled as "2" with the appropriate factorization of fermion mass terms. For the full SM corrections one must include all diagrams since the photon cannot be separated from the other gauge bosons in a gauge invariant way

[2] Note that here we only consider the fermion mass singular terms. For the full SM factorization of SL- logarithmic terms, all diagrams need to be calculated since the photon cannot be separated from the other gauge bosons in a gauge invariant way

schematically depicted in the figure. As mentioned above, the exchange of the heavy gauge bosons does not lead to fermion mass singular terms. Taking into account all diagrams of the SM leads, however, to uncancelled $\log^{2n}(s/M^2)$ Bloch–Nordsieck violating terms in fully inclusive cross sections since the initial state carries isospin [17].

The scalar sector in itself, after the application of the equivalence theorem, poses no new problem since the angular SL terms originate from soft corrections. In [9,8] the universal nature of Yukawa enhanced SL corrections was investigated and all higher order SL terms resummed in [8]. By direct inspection of the full electroweak Feynman rules it can easily be seen that there are no angle dependent SL corrections with Yukawa terms possible which are not mass suppressed. Thus, whether we consider fundamental fermions or scalars charged under the unbroken gauge group leads to the same form of factorized corrections.

A more serious problem, in principle, is given by the mass gap of the gauge bosons, since there could be novel soft angular corrections compared to the equal mass case. This, however, is not the case since the soft angular terms from virtual scattering amplitudes in the full SM are fixed by the pure real QED corrections in the regime where the experimental energy resolution ΔE is below the weak scale M. This energy cut provides a well defined observable in the SM without the inclusion of real heavy gauge boson emission. Since the real QED corrections are always proportional to $\log(2\Delta E/\lambda)$ at one loop, and exponentiates at higher orders, the infrared finiteness of observable cross sections fixes the type of soft virtual angular corrections to that of pure QED effects. In addition we have seen for the case of right handed massive fermions in (10) explicitly at the two loop level that the gauge boson mass gap does not spoil the factorization of the virtual SL corrections and therefore leads to the SL exponentation.

The next to leading order resummation of QCD corrections was first derived for on-shell quark scattering in [18] and important aspects are also discussed in [19,20].

The general structure of pole terms for virtual two loop massless QCD scattering amplitudes was presented in [21] and subsequently confirmed by explicit two loop calculations in [22,23] in two to two processes. While the result presented in [21] has been formulated for $\overline{\mathrm{MS}}$ renormalized on-shell amplitudes, the form of the factorization is regularization scheme independent at least up to single pole terms. An application of the factorization formula in the QED limit to elastic Bhabha scattering was presented in [24]. It uses mass regulators for the soft and collinear divergences. To SL accuracy in QED, one loop results calculated in this mass regulator scheme determine the corresponding two loop logarithmic corrections.

Also in the general case, a main feature of the factorization formula at the two loop level is that – up to terms corresponding to higher order running coupling terms – all leading and subleading pole terms are determined by the one loop divergences in color space. The latter was formulated in [25] and serves as a convenient way to incorporate soft color correlations in QCD scattering amplitudes which were also discussed in [26]. These are novel effects in non-Abelian gauge theories and need to be properly included.

In the electroweak theory, the corresponding correlations are due to the exchange of the $W^{\pm}$ only since the Z-boson and the photon do not change the flavor. Thus, it is legitimate for these terms to only consider a theory above the scale M, i.e. an unbroken massless $SU(2)\times U(1)$ theory with all gauge bosons regularized by the same mass M. This theory then has angular corrections in the same form as QCD, now however, formulated in the n-particle space.

From the above arguments it is now clear that to SL accuracy, neglecting RG–SL terms for now[3], the virtual SM corrections in the on-shell scheme at the one and two loop level for processes with n external lines can be expressed as

$$\begin{aligned}
&\mathcal{M}_{(1)}^{i_1,\dots,i_n}(p_1,...,p_n;m_1,...,m_n;M,\lambda) \\
&\quad = I_n^{(1)}(\{p_k\},\{m_l\},M,\lambda) \\
&\quad \times \mathcal{M}_{\mathrm{Born}}^{i_1,\dots,i'_k,\dots,i'_l,\dots,i_n}(p_1,...,p_n;m_1,...,m_n), \\
&\mathcal{M}_{(2)}^{i_1,\dots,i_n}(p_1,...,p_n;m_1,...,m_n;M,\lambda) \\
&\quad = \frac{1}{2} I_n^{(1)}(\{p_k\},\{m_l\},M,\lambda)\, I_n^{(1)}(\{p_k\},\{m_l\},M,\lambda) \\
&\quad \times \mathcal{M}_{\mathrm{Born}}^{i_1,\dots,i'_k,\dots,i'_l,\dots,i_n}(p_1,...,p_n;m_1,...,m_n). \qquad (12)
\end{aligned}$$

All finite terms at one loop and the corresponding terms at two loops below SL accuracy are omitted in (12). The n-particle space operator $I_n^{(1)}(\{p_k\},\{m_l\},M,\lambda)$ is determined by the one loop structure and its explicit form is given by

$$\begin{aligned}
I_n^{(1)}(\{p_k\},\{m_l\},M,\lambda) &= \frac{e^2}{16\pi^2}\sum_{k=1}^{n}\Bigg\{-\frac{1}{2}\Bigg[C^{\mathrm{ew}}_{i'_k,i_k}\log^2\frac{s}{M^2} \\
&+ \delta_{i'_k,i_k}Q_k^2\left(2\log\frac{s}{M^2}\log\frac{M^2}{\lambda^2}+\log^2\frac{M^2}{\lambda^2}-\log^2\frac{m_k^2}{\lambda^2}\right)\Bigg] \\
&+ \delta^{\mathrm{SL}}_{i'_k,i_k}\log\frac{s}{M^2} \\
&+ 2\sum_{l<k}^{n}\sum_{V_a=A,Z,W^{\pm}} I^{V_a}_{i'_k,i_k} I^{\overline{V}_a}_{i'_l,i_l}\log\frac{s}{m^2_{V_a}}\log\frac{2p_lp_k}{s}\Bigg\}, \qquad (13)
\end{aligned}$$

where all lines are assumed as incoming, the symbols $I^{V_a}_{i'_k,i_k}$ denote the couplings of the $\overline{\varphi}_{i_k}V_a\varphi_{i'_k}$ vertices and $C^{\mathrm{ew}}_{i'_k,i_k}$ the corresponding electroweak eigenvalue of the Casimir operator. The DL corrections were first obtained in [1] and for longitudinal polarizations and Higgs final states in [7], in both cases actually to all orders. The general form of the angle dependent corrections at one loop was first derived in [9]. The subleading universal corrections $\delta^{\mathrm{SL}}_{i'_k,i_k}$ depend on the external line only and are given in [9,

[3] These terms are discussed in [27], and are related to external lines only. Thus they do not depend on angular terms and are therefore not important for the current discussion. The exponentiation of the universal terms was discussed in [1,7,8]

7,8]. For massless four fermion processes, the SL angular and universal SL terms were first derived in [11] to all orders.

In the non-diagonal part of (13) it can be seen that in distinction to an unbroken gauge theory, the angular terms include quark mixing effects (included in the couplings $I^{V_a}_{i'_k,i_k}$) and depend on different masses, i.e. the masses of the electroweak gauge bosons. Due to our discussion above, however, fermion mass singularities are fixed by the KLN theorem and the soft angular terms of QED origin by the infrared finiteness of semi-inclusive cross sections.

3 Angular dependent corrections in the effective theory

In this section we generalize the two loop results of the previous section in (12) to all orders by reinterpreting the angular terms in the language of the high energy effective theory based on the Lagrangian in the symmetric basis. The physical picture is that of the approximately restored $SU_L(2) \times U_Y(1)$ gauge symmetry at high energies where terms with a mass dimension can be neglected. The soft QED effects below the scale M, set by the electroweak gauge bosons, can be included via matching at that scale. For a thorough discussion see [1,2].

Amplitudes in terms of the physical fields (f) are given in terms of superpositions of the fields in the unbroken phase (u) as follows [2]:

$$\begin{aligned}&\mathcal{M}^{f_1,\ldots,f_n}(\{p_k\};\{m_l\};M,\lambda)\\&\quad=\sum_{u_1,\ldots,u_n}\prod_{j=1}^{n}C^{f_j u_j}\mathcal{M}^{u_1,\ldots,u_n}(\{p_k\};\{m_l\};M,\lambda),\end{aligned} \tag{14}$$

where the $C^{f_j u_j}$ denote the corresponding mixing coefficients. We note that at one loop, also the renormalization conditions involving the mixing coefficients need to be included properly as discussed in [7,9]. The fields u have a well defined isospin, but for angle dependent terms involving CKM mixing effects, one has to include the extended isospin mixing appropriately in the corresponding couplings $\tilde{I}^{V_a}_{i'_k,i_k}$. In the following we give the corrections only for the amplitudes $\mathcal{M}^{u_1,\ldots,u_n}(\{p_k\};\{m_l\};M,\lambda)$.

As far as the angle dependent terms are concerned, it is clear that above the scale M, i.e. for a photon with mass M rather than λ, the difference to the corresponding result in (12) is given by a change of basis, i.e.

$$\begin{aligned}&\sum_{k=1}^{n}\sum_{l<k}^{n}\sum_{V_a=A,Z,W^\pm}e^2 I^{V_a}_{i'_k,i_k}I^{\overline{V}_a}_{i'_l,i_l}\log\frac{s}{M^2}\\&\times\log\frac{2p_lp_k}{s}\mathcal{M}_{\mathrm{Born}}^{f_{i_1},\ldots,f_{i'_k},\ldots,f_{i'_l},\ldots,f_{i_n}}(\{p_k\};\{m_l\})\\&=\sum_{u_1,\ldots u_n}\prod_{j=1}^{n}C^{f_ju_j}\sum_{k=1}^{n}\sum_{l<k}^{n}\sum_{V_a=B,W^a}\tilde{I}^{V_a}_{i'_k,i_k}\tilde{I}^{\overline{V}_a}_{i'_l,i_l}\log\frac{s}{M^2}\\&\times\log\frac{2p_lp_k}{s}\mathcal{M}_{\mathrm{Born}}^{u_{i_1},\ldots,u_{i'_k},\ldots,u_{i'_l},\ldots,u_{i_n}}(\{p_k\};\{m_l\}).\end{aligned} \tag{15}$$

The remaining terms are then just given by QED corrections of the type

$$\sum_{k=1}^{n}\sum_{l<k}^{n}Q_kQ_l\log\frac{M^2}{\lambda^2}\log\frac{2p_lp_k}{s}. \tag{16}$$

The terms in (16) therefore correspond precisely to matching terms in analogy to the situation for the universal SL logarithms [7,8]. Thus, if $\mathcal{L}_{\mathrm{symm}}$ is a valid effective theory at high energies in the sense that it contains all relevant physical degrees of freedom[4], then matching at the scale M must yield the corresponding soft QED terms since these are fixed by the real QED corrections and their higher order behavior.

The arguments given in Sect. 2 with respect to the (non-) influence of spontaneous symmetry breaking on the form of the SL angle dependent corrections at the two loop level can easily be generalized to arbitrary order in perturbation theory since the infrared finiteness of cross sections and with it the exponentiation of the real soft bremsstrahlung corrections holds to all orders as does the KLN theorem, the equivalence theorem in the longitudinal sector and the non-existence of Yukawa dependent angular SL terms. The associated exponentiation of the non-Abelian high energy regime is now in matrix form analogous to QCD [21] and reads for the purely virtual corrections in the on-shell scheme as follows[5]:

$$\begin{aligned}&\mathcal{M}_{\mathrm{SL}}^{u_{i_1},\ldots,u_{i_n}}(\{p_k\};\{m_l\};M,\lambda)\\&=\exp\Bigg\{-\frac{1}{2}\sum_{i=1}^{n_g}W^{\mathrm{RG}}_{g_i}(s,M^2)-\frac{1}{2}\sum_{i=1}^{n_f}W^{\mathrm{RG}}_{f_i}(s,M^2)\\&-\frac{1}{2}\sum_{i=1}^{n_\phi}W^{\mathrm{RG}}_{\phi_i}(s,M^2)+\frac{1}{8\pi^2}\sum_{k=1}^{n}\sum_{l<k}^{n}\sum_{V_a=B,W^a}\tilde{I}^{V_a}_{i'_k,i_k}\tilde{I}^{\overline{V}_a}_{i'_l,i_l}\\&\times\log\frac{s}{M^2}\log\frac{2p_lp_k}{s}\Bigg\}\\&\times\exp\Bigg[-\frac{1}{2}\sum_{i=1}^{n_f}\left(w^{\mathrm{RG}}_{f_i}(s,\lambda^2)-w^{\mathrm{RG}}_{f_i}(s,M^2)\right)\\&-\frac{1}{2}\sum_{i=1}^{n_{\mathrm{w}}}\left(w^{\mathrm{RG}}_{\mathrm{w}_i}(s,\lambda^2)-w^{\mathrm{RG}}_{\mathrm{w}_i}(s,M^2)\right)-\frac{1}{2}\sum_{i=1}^{n_\gamma}w_{\gamma_i}(M^2,m_j^2)\\&+\sum_{k=1}^{n}\sum_{l<k}^{n}\left(w^{\theta}_{kl}\left(s,\lambda^2\right)-w^{\theta}_{kl}\left(s,M^2\right)\right)\Bigg]\\&\times\mathcal{M}_{\mathrm{Born}}^{u_{i_1},\ldots,u_{i'_k},\ldots,u_{i'_l},\ldots,u_{i_n}}(\{p_k\};\{m_l\}),\end{aligned} \tag{17}$$

[4] For the angle dependent corrections in the massive quark sector we must in principle include the CKM terms in the corresponding couplings as mentioned above

[5] In this paper we use a photon mass λ regulator for the soft QED effects in order to simplify the comparisons with calculations using the physical fields. This is identical to using a cutoff in the exchanged perpendicular components of the emitted particles if the cutoff is much smaller than all other parameters in the theory. If one wants to check the validity of the matching conditions one has to use the general form given in [1,2]

where n_g denotes the number of external gauge bosons (in the symmetric basis), n_f the number of external fermions and n_ϕ the number of external scalars (including Higgs particles). The notation used in the regime below the scale M is analogous. The diagonal terms do not involve (CKM extended) isospin rotated Born matrix elements. These occur only from the angular terms above the scale M. At the two loop level, (17) represents the novel result of this work. It agrees in the massless limit for four fermion processes with the results presented in [11] and reproduces (12) in the two loop expansion after using (14) up to RG-SL terms which we included here. It should be noted that (17) is valid for arbitray processes and number of participating particles. The explicit expressions for the various ingredients given in (17) are given by

$$W^{\rm RG}_{\phi_i}(s,M^2) = \frac{\alpha(M^2)T_i(T_i+1)}{2\pi} \times \left\{ \frac{1}{c} \log \frac{s}{M^2} \left(\log \frac{\alpha(M^2)}{\alpha(s)} - 1 \right) + \frac{1}{c^2} \log \frac{\alpha(M^2)}{\alpha(s)} \right\}, + \frac{\alpha'(M^2)Y_i^2}{8\pi} \left\{ \frac{1}{c'} \log \frac{s}{M^2} \left(\log \frac{\alpha'(M^2)}{\alpha'(s)} - 1 \right) + \frac{1}{c'^2} \log \frac{\alpha'(M^2)}{\alpha'(s)} \right\} - \left[\left(\frac{\alpha(M^2)}{4\pi} T_i(T_i+1) + \frac{\alpha'(M^2)}{4\pi} \frac{Y_i^2}{4} \right) 4 \log \frac{s}{M^2} - \frac{3}{2} \frac{\alpha(M^2)}{4\pi} \frac{m_t^2}{M^2} \log \frac{s}{m_t^2} \right], \quad (18)$$

where T_i denotes the total weak isospin of particle i and Y_i its hypercharge. Furthermore, we denote the $SU(2)$ coupling by $\alpha = g^2/4\pi$ and that of $U(1)$ by $\alpha' = {g'}^2/4\pi$. The two one loop contributions to the respective β-functions are given by

$$\beta_0 = \frac{11}{12} C_A - \frac{1}{3} n_{\rm gen} - \frac{1}{24} n_h$$

and

$$\beta_0' = -\frac{5}{9} n_{\rm gen} - \frac{1}{24} n_h,$$

where $n_{\rm gen}$ denotes the number of generations and n_h the number of Higgs doublets. We can use m_t in the logarithmic argument of the Yukawa enhanced correction [8]. In addition we denote $c = g^2/4\pi^2\beta_0$ and $c' = {g'}^2/4\pi^2\beta_0'$. Analogously for fermions we have

$$W^{\rm RG}_{f_i}(s,M^2) = \frac{\alpha(M^2)T_i(T_i+1)}{2\pi} \times \left\{ \frac{1}{c} \log \frac{s}{M^2} \left(\log \frac{\alpha(M^2)}{\alpha(s)} - 1 \right) + \frac{1}{c^2} \log \frac{\alpha(M^2)}{\alpha(s)} \right\} + \frac{\alpha'(M^2)Y_i^2}{8\pi} \times \left\{ \frac{1}{c'} \log \frac{s}{M^2} \left(\log \frac{\alpha'(M^2)}{\alpha'(s)} - 1 \right) + \frac{1}{c'^2} \log \frac{\alpha'(M^2)}{\alpha'(s)} \right\} - \left[\left(\frac{\alpha(M^2)}{4\pi} T_i(T_i+1) + \frac{\alpha'(M^2)}{4\pi} \frac{Y_i^2}{4} \right) 3 \log \frac{s}{M^2} - \frac{\alpha(M^2)}{4\pi} \left(\frac{1+\delta_{i,\rm R}}{4} \frac{m_i^2}{M^2} + \delta_{i,\rm L} \frac{m_{i'}^2}{4M^2} \right) \log \frac{s}{m_t^2} \right]. \quad (19)$$

The last term contributes only for left handed bottom as well as for top quarks and f' denotes the corresponding isospin partner for left handed fermions.

$$W^{\rm RG}_{g_i}(s,M^2) = \frac{\alpha(M^2)T_i(T_i+1)}{2\pi} \times \left\{ \frac{1}{c} \log \frac{s}{M^2} \left(\log \frac{\alpha(M^2)}{\alpha(s)} - 1 \right) + \frac{1}{c^2} \log \frac{\alpha(M^2)}{\alpha(s)} \right\} + \frac{\alpha'(M^2)Y_i^2}{8\pi} \times \left\{ \frac{1}{c'} \log \frac{s}{M^2} \left(\log \frac{\alpha'(M^2)}{\alpha'(s)} - 1 \right) + \frac{1}{c'^2} \log \frac{\alpha'(M^2)}{\alpha'(s)} \right\} - \left(\delta_{i,\rm W} \frac{\alpha(M^2)}{\pi} \beta_0 + \delta_{i,\rm B} \frac{\alpha'(M^2)}{\pi} \beta_0' \right) \log \frac{s}{M^2}. \quad (20)$$

We note that for external photon and Z-boson states we must include the mixing appropriately as discussed in [7]. For the terms entering from contributions below the weak scale we have for fermions:

$$w^{\rm RG}_{f_i}(s,\lambda^2) = \frac{e_i^2}{8\pi^2} \times \left\{ \frac{1}{c} \log \frac{s}{m^2} \left(\log \frac{e^2(\lambda^2)}{e^2(s)} - 1 \right) - \frac{3}{2} \log \frac{s}{m^2} - \log \frac{m^2}{\lambda^2} + \frac{1}{c^2} \log \frac{e^2(m^2)}{e^2(s)} \left(1 - \frac{1}{3} \frac{e^2}{4\pi^2} \sum_{j=1}^{n_f} Q_j^2 N_C^j \log \frac{m^2}{m_j^2} \right) \right\}, \quad (21)$$

where $c = -(1/3)(e^2/4\pi^2) \sum_{j=1}^{n_f} Q_j^2 N_C^j$ and the running QED coupling given in (33) and $e_i \equiv eQ_i$. Analogously, for external W-bosons and photons we find:

$$w^{\rm RG}_{{\rm w}_i}(s,\lambda^2) = \frac{e_i^2}{8\pi^2} \times \left\{ \frac{1}{c} \log \frac{s}{M^2} \left(\log \frac{e^2(\lambda^2)}{e^2(s)} - 1 \right) - \log \frac{M^2}{\lambda^2} + \frac{1}{c^2} \log \frac{e^2(M^2)}{e^2(s)} \left(1 - \frac{1}{3} \frac{e^2}{4\pi^2} \sum_{j=1}^{n_f} Q_j^2 N_C^j \log \frac{M^2}{m_j^2} \right) \right\}, \quad (22)$$

$$w_{\gamma_i}(M^2, m_j^2) = \frac{1}{3} \sum_{j=1}^{n_f} \frac{e_j^2}{4\pi^2} N_C^j \log \frac{M^2}{m_j^2}. \quad (23)$$

Note that $w_{\gamma_i}(M^2,M^2) = 0$. Finally, we have

$$w^\theta_{kl}\left(s,\lambda^2\right) = \frac{e^2}{8\pi^2} Q_k Q_l \log \frac{s}{\lambda^2} \log \frac{2p_l p_k}{s}, \quad (24)$$

for the angle dependent corrections from the soft QED regime.

In distinction to the universal terms, the angle dependent corrections in (17) are not related to the emission probabilities for soft and/or collinear gauge bosons. As mentioned above, the fundamental objects in the high energy regime above the scale M are the fields in $\mathcal{L}_{\rm symm}$ as described in [2]. In order to clarify the method, we will compare the predictions in the next section starting at the one loop level.

4 Comparison with known results

In this section we will compare our approach with existing calculations. While the presented results are known[6], it serves to illustrate the method and demonstrates the powerful constraints given by real soft emissions as well as the overall check of the high energy effective theory. In particular we would like to emphasize that for the universal corrections the splitting function approach of [7, 8] predicts that the factorization of DL and SL terms for fermions and scalars takes place with the same electroweak group factor

$$-\frac{1}{2}\left(\frac{g^2}{16\pi^2}T(T+1)+\frac{g'^2}{16\pi^2}\frac{Y^2}{4}\right).$$

Only the Yukawa terms and transverse gauge boson anomalous dimensions factorize differently, namely with

$$-(g^2/32\pi^2)(3/2)m_t^2/M^2 \text{ and } (g^2/8\pi^2)\beta_0 \left((g'^2/8\pi^2)\beta_0'\right)$$

correspondingly.

The results of [28] were obtained using the physical fields. Soft real photon radiation will be included in the comparison. In the following, the lower index on the cross section indicates the helicity of the electron, where e^-_- denotes the left handed electron. We summarize the relevant results for $e^+_- e^-_+ \longrightarrow W^+_{\rm T} W^-_{\rm T}$, $e^+_+ e^-_- \longrightarrow W^+_{\rm L} W^-_{\rm L}$ and $e^+_- e^-_+ \longrightarrow W^+_{\rm L} W^-_{\rm L}$ from [28] for convenience as follows:

$$\begin{aligned}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{-,\mathrm{T}} &\approx \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\rm Born}_{-,\mathrm{T}} \\ &\times \left\{1+\frac{e^2}{8\pi^2}\left[-\frac{1+2c_{\rm w}^2+8c_{\rm w}^4}{4c_{\rm w}^2 s_{\rm w}^2}\log^2\frac{s}{M^2}\right.\right. \\ &+3\frac{1-2c_{\rm w}^2+4c_{\rm w}^4}{4c_{\rm w}^2 s_{\rm w}^2}\log\frac{s}{M^2}+\frac{4u+2s}{s_{\rm w}^2 u}\log\frac{s}{M^2}\log\frac{s}{-t} \\ &-\frac{2(1-2c_{\rm w}^2)}{s_{\rm w}^2}\log\frac{s}{M^2}\log\frac{u}{t}+3\log\frac{s}{m_e^2}, \\ &+2\log\frac{4\Delta E^2}{s}\left(\log\frac{s}{m_e^2}+\log\frac{s}{M^2}+2\log\frac{t}{u}-2\right) \\ &\left.\left.-\frac{4}{3}\sum_{j=1}^{n_f}Q_j^2 N_C^j\log\frac{m_j^2}{M^2}\right]\right\},\end{aligned} \tag{25}$$

$$\begin{aligned}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{-,\mathrm{L}} &\approx \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\rm Born}_{-,\mathrm{L}} \\ &\times \left\{1+\frac{e^2}{8\pi^2}\left[-\frac{1-2c_{\rm w}^2+4c_{\rm w}^4}{2c_{\rm w}^2 s_{\rm w}^2}\log^2\frac{s}{M^2}\right.\right. \\ &+\frac{103-158c_{\rm w}^2+80c_{\rm w}^4}{12c_{\rm w}^2 s_{\rm w}^2}\log\frac{s}{M^2} \\ &-\frac{3m_t^2}{2s_{\rm w}^2 M^2}\log\frac{s}{m_t^2}+\frac{4c_{\rm w}^2}{s_{\rm w}^2}\log\frac{s}{M^2}\log\frac{s}{-t} \\ &+\frac{(1-2c_{\rm w}^2)^2}{c_{\rm w}^2 s_{\rm w}^2}\log\frac{s}{M^2}\log\frac{u}{t}+3\log\frac{s}{m_e^2}, \\ &+2\log\frac{4\Delta E^2}{s}\left(\log\frac{s}{m_e^2}+\log\frac{s}{M^2}+2\log\frac{t}{u}-2\right) \\ &\left.\left.-\frac{4}{3}\sum_{j=1}^{n_f}Q_j^2 N_C^j\log\frac{m_j^2}{M^2}\right]\right\},\end{aligned} \tag{26}$$

$$\begin{aligned}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{+,\mathrm{L}} &\approx \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\rm Born}_{+,\mathrm{L}} \\ &\times \left\{1+\frac{e^2}{8\pi^2}\left[-\frac{5-10c_{\rm w}^2+8c_{\rm w}^4}{4c_{\rm w}^2 s_{\rm w}^2}\log^2\frac{s}{M^2}\right.\right. \\ &+\frac{65-65c_{\rm w}^2+18c_{\rm w}^4}{6c_{\rm w}^2 s_{\rm w}^2}\log\frac{s}{M^2}-\frac{3m_t^2}{2s_{\rm w}^2 M^2}\log\frac{s}{m_t^2} \\ &+\frac{2(1-2c_{\rm w}^2)}{c_{\rm w}^2}\log\frac{s}{M^2}\log\frac{u}{t}+3\log\frac{s}{m_e^2}, \\ &+2\log\frac{4\Delta E^2}{s}\left(\log\frac{s}{m_e^2}+\log\frac{s}{M^2}+2\log\frac{t}{u}-2\right) \\ &\left.\left.-\frac{4}{3}\sum_{j=1}^{n_f}Q_j^2 N_C^j\log\frac{m_j^2}{M^2}\right]\right\},\end{aligned} \tag{27}$$

where at high energies we denote $t=-(s/2)(1-\cos\theta)$ and $u=-(s/2)(1+\cos\theta)$. The angle θ is the one between the incoming e^+ and the outgoing W^+. The Born cross sections are given by

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\rm Born}_{-,\mathrm{T}} = \frac{e^4}{64\pi^2 s}\frac{1}{4s_{\rm w}^4}\frac{u^2+t^2}{t^2}\sin^2\theta, \tag{28}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\rm Born}_{-,\mathrm{L}} = \frac{e^4}{64\pi^2 s}\frac{1}{16s_{\rm w}^4 c_{\rm w}^4}\sin^2\theta, \tag{29}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\rm Born}_{+,\mathrm{L}} = \frac{e^4}{64\pi^2 s}\frac{1}{4c_{\rm w}^4}\sin^2\theta. \tag{30}$$

In (28) a sum over the two transverse polarizations of the $W^\pm$ ($++$ and $--$) is implicit. These expressions demonstrate that the longitudinal cross sections in (29) and (30) are not mass suppressed (while $(\mathrm{d}\sigma/\mathrm{d}\Omega)^{\rm Born}_{+,\mathrm{T}}$ is). Equations (25), (26) and (27) were of course calculated in terms of the physical fields of the broken theory and in the on-shell scheme. We denote $c_{\rm w}=\cos\theta_{\rm w}$ and $s_{\rm w}=\sin\theta_{\rm w}$ respectively.

[6] At one loop, general results for all SL corrections are given in [9] using the physical SM fields

In the above results we have included soft bremsstrahlung contributions. In order to demonstrate how the soft angle dependent real QED corrections serve to fix the virtual angular terms at the one loop level (and in general also the higher order terms) we list the one loop purely real corrections separately. To SL accuracy we have for the above cross sections in the limit $k_0 \leq \Delta E \leq M$:

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{brems}} \approx \left(\frac{d\sigma}{d\Omega}\right)^{\text{Born}} \left(-\frac{e^2}{4\pi^2}\right) \times \left\{ 4\log\frac{2\Delta E}{\lambda} - 2\log\frac{2\Delta E}{\lambda}\log\frac{s}{m_e^2} + 4\log\frac{2\Delta E}{\lambda}\log\frac{u}{t} -2\log\frac{2\Delta E}{\lambda}\log\frac{s}{M^2} + \frac{1}{2}\log^2\frac{s}{m_e^2} - \log\frac{s}{m_e^2} + \frac{1}{2}\log^2\frac{s}{M^2} - \log\frac{s}{M^2} \right\}. \tag{31}$$

It can be seen in (31) that all angle dependent soft terms are proportional to $\log(2\Delta E/\lambda)$, which means that there is no freedom for the virtual angular terms in order to avoid the infrared divergence as $\lambda \to 0$. Note also that all soft bremsstrahlung corrections are not sensitive to the spin of the final state as expected (we use the equivalence theorem such that the longitudinal degrees of freedom are described by scalar particles).

Using

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad s_{\text{w}} = \frac{g'}{\sqrt{g^2 + g'^2}}$$

and

$$c_{\text{w}} = \frac{g}{\sqrt{g^2 + g'^2}},$$

we see that the Born cross section in (28) is proportional to g^4, in (29) proportional to $(g^2 + g'^2)^2$ and (30) proportional to g'^4. Thus, in the transverse sector the coupling renormalization above the scale M is given by

$$g^2(s) = g^2(M^2) \times \left(1 - \frac{g^2(M^2)}{4\pi^2}\left(\frac{11}{12}C_A - \frac{1}{24}n_h - \frac{n_{\text{gen}}}{3}\right)\log\frac{s}{M^2}\right) = \frac{e^2_{\text{eff}}(M^2)}{s^2_{\text{w}}}\left(1 - \frac{e^2_{\text{eff}}(M^2)}{4\pi^2 s^2_{\text{w}}}\frac{19}{24}\log\frac{s}{M^2}\right), \tag{32}$$

where in the second line we use $C_A = 2$, $n_{\text{gen}} = 3$ and $n_h = 1$. Below the scale where non-Abelian effects enter, the running is only due to the electromagnetic coupling with

$$e^2_{\text{eff}}(M^2) = e^2\left(1 + \frac{1}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f} Q_j^2 N_C^j \log\frac{M^2}{m_j^2}\right), \tag{33}$$

with $e^2/4\pi = 1/137$. The form of (33) is purely perturbative and does not correctly reproduce the non-perturbative hadronic contribution. We observe that the running coupling terms proportional to $\log s/M^2$ cancel for the process involving transverse gauge bosons with the subleading contributions from the virtual splitting functions at one loop (see (20)) and what remains are just the Abelian terms up to scale M.

The RG corrections to both longitudinal cross sections accounting for the running couplings from M^2 to s are given by

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{RG}}_{-,\text{L}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{Born}}_{-,\text{L}} \times \left\{1 + \frac{e^2}{8\pi^2}\frac{41 - 82c^2_{\text{w}} + 22c^4_{\text{w}}}{6s^2_{\text{w}}c^2_{\text{w}}}\log\frac{s}{M^2}\right\}, \tag{34}$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{RG}}_{+,\text{L}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{Born}}_{+,\text{L}} \left\{1 + \frac{e^2}{8\pi^2}\frac{41}{6c^2_{\text{w}}}\log\frac{s}{M^2}\right\}. \tag{35}$$

In order to account for the angle dependent parts in (25), (26) and (27) we need to write the amplitude with isospin rotated Born matrix elements (in the symmetric basis above the scale M) as corrections proportional to the physical Born amplitude. This is always possible since we are dealing with functions of the invariants only in the high energy limit. In this way we have, using a regulator mass $M \geq m_i$, the following contributions relative to the Born amplitude:

$$\sum_{B,W^a} \delta^\theta_{e^+_+ e^-_- \longrightarrow W^+_{\text{T}} W^-_{\text{T}}} = -\frac{g^2}{8\pi^2}\log\frac{s}{M^2}\left(\log\frac{t}{u} + \left(1 - \frac{t}{u}\right)\log\frac{-t}{s}\right), \tag{36}$$

$$\sum_{B,W^a} \delta^\theta_{e^+_+ e^-_- \longrightarrow \phi^+\phi^-} = -\frac{g^2}{8\pi^2}\log\frac{s}{M^2}\left(\frac{1}{2c^2_{\text{w}}}\log\frac{t}{u} + 2c^2_{\text{w}}\log\frac{-t}{s}\right), \tag{37}$$

$$\sum_{B,W^a} \delta^\theta_{e^+_- e^-_+ \longrightarrow \phi^+\phi^-} = -\frac{g'^2}{8\pi^2}\log\frac{s}{M^2}\log\frac{t}{u}. \tag{38}$$

In addition we have the soft angular contributions which are given by

$$\sum_{kl} w^\theta_{kl}(s,\lambda) = -\frac{e^2}{4\pi^2}\log\frac{s}{\lambda^2}\log\frac{t}{u}, \tag{39}$$

which is again independent of the spin plus the corresponding matching term (see (17)). The Sudakov corrections to both cross sections from the infrared evolution equation method according to (17) in the soft photon approximation are given below. The quantum numbers are those of the particle indices and are summarized in Table 1. We have

$$\left(\frac{d\sigma}{d\Omega}\right)_{-,\text{T}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{Born}}_{-,\text{T}} \times \left\{1 - \left(\frac{g^2}{8\pi^2}T_{\text{w}}(T_{\text{w}}+1) + \frac{g'^2}{8\pi^2}\frac{Y^2_{\text{w}}}{4}\right)\log^2\frac{s}{M^2} - \left(\frac{g^2}{8\pi^2}T_{e^-_-}(T_{e^-_-}+1) + \frac{g'^2}{8\pi^2}\frac{Y^2_{e^-_-}}{4}\right)\right.$$

Table 1. The quantum numbers of various particles in the electroweak theory. The indices indicate the helicity of the electrons and quarks. In the high energy regime described by the Lagrangian in the symmetric basis, we neglect all mass terms, i.e. we consider all particles as chiral eigenstates with well defined total weak isospin (T) and weak hypercharge (Y) quantum numbers (except for the photon and the Z-boson as discussed in the text). In each case, the electric charge Q, measured in units of the proton charge, by the Gell-Mann–Nishijima formula $Q = T^3 + Y/2$. For longitudinally polarized gauge bosons, the associated scalar Goldstone bosons describe the DL and SL asymptotics

	T	Y	Q
e^-_-	1/2	-1	-1
e^-_+	0	-2	-1
e^+_+	1/2	1	1
e^+_-	0	2	1
u_-	1/2	1/3	2/3
u_+	0	4/3	2/3
d_-	1/2	1/3	-1/3
d_+	0	-2/3	-1/3
$W^\pm_{\mathrm{T}}$	1	0	± 1
$\phi^\pm$	1/2	± 1	± 1
χ	1/2	+1	0
H	1/2	+1	0

$$
\begin{aligned}
&\times \left(\log^2 \frac{s}{M^2} - 3\log\frac{s}{M^2}\right) \\
&-\frac{g^2}{4\pi^2}\log\frac{s}{M^2}\left(\log\frac{t}{u} + \left(1-\frac{t}{u}\right)\log\frac{-t}{s}\right) \\
&-\frac{e^2}{8\pi^2}\left[\left(\log\frac{s}{m_e^2}-1\right)2\log\frac{m_e^2}{\lambda^2} + \log^2\frac{s}{m_e^2}\right. \\
&-3\log\frac{s}{m_e^2} - \log^2\frac{s}{M^2} + 3\log\frac{s}{M^2} \\
&+2\left(\log\frac{s}{M^2}-1\right)\log\frac{M^2}{\lambda^2} \\
&-\left(\log\frac{s}{m_e^2}-1\right)\left(2\log\frac{4(\Delta E)^2}{\lambda^2} - 2\log\frac{s}{m_e^2}\right), \\
&-2\left(\log\frac{s}{M^2}-1\right)\left(\log\frac{4(\Delta E)^2}{\lambda^2} - \log\frac{s}{M^2}\right) \\
&\left.-\log^2\frac{s}{m_e^2} - \log^2\frac{s}{M^2}\right] \\
&\left.+\frac{e^2}{2\pi^2}\log\frac{4(\Delta E)^2}{M^2}\log\frac{t}{u} + \frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f} Q_j^2 N_C^j \log\frac{M^2}{m_j^2}\right\} \\
&= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{Born}}_{-,\mathrm{T}} \\
&\times\left\{1 - \frac{e^2}{8\pi^2}\left[\frac{1+10c_{\mathrm{w}}^2}{4s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log^2\frac{s}{M^2} - 3\frac{1+2c_{\mathrm{w}}^2}{4s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log\frac{s}{M^2}\right.\right. \\
&+\frac{2}{s_{\mathrm{w}}^2}\log\frac{s}{M^2}\left(\left(2c_{\mathrm{w}}^2-1\right)\log\frac{t}{u} + \left(1-\frac{t}{u}\right)\log\frac{-t}{s}\right) \\
&-2\log^2\frac{s}{M^2} + 3\log\frac{m_e^2}{M^2} \\
&\left.+4\log\frac{s}{4(\Delta E)^2}\left(\log\frac{s}{m_e M} + \log\frac{t}{u} - 1\right)\right] \\
&\left.+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f} Q_j^2 N_C^j \log\frac{M^2}{m_j^2}\right\}, \qquad (40)
\end{aligned}
$$

which reproduces the correct result of (25). For longitudinal degrees of freedom we use the equivalence theorem and find analogously

$$
\begin{aligned}
&\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{-,\mathrm{L}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{Born}}_{-,\mathrm{L}} \\
&\times\left\{1 - \left(\frac{g^2}{8\pi^2}T_\phi(T_\phi+1) + \frac{g'^2}{8\pi^2}\frac{Y_\phi^2}{4}\right)\right. \\
&\times\left(\log^2\frac{s}{M^2} - 4\log\frac{s}{M^2}\right) \\
&-\left(\frac{g^2}{8\pi^2}T_{e^-_-}(T_{e^-_-}+1) + \frac{g'^2}{8\pi^2}\frac{Y_{e^-_-}^2}{4}\right) \\
&\times\left(\log^2\frac{s}{M^2} - 3\log\frac{s}{M^2}\right) \\
&-\frac{g^2}{4\pi^2}\log\frac{s}{M^2}\left(\frac{1}{2c_{\mathrm{w}}^2}\log\frac{t}{u} + 2c_{\mathrm{w}}^2\log\frac{-t}{s}\right) \\
&-3\frac{g^2}{16\pi^2}\frac{m_t^2}{M^2}\log\frac{s}{m_t^2} - \frac{e^2}{8\pi^2}\left[\left(\log\frac{s}{m_e^2}-1\right)2\log\frac{m_e^2}{\lambda^2}\right. \\
&+\log^2\frac{s}{m_e^2} - 3\log\frac{s}{m_e^2} - \log^2\frac{s}{M^2} + 3\log\frac{s}{M^2} \\
&+2\left(\log\frac{s}{M^2}-1\right)\log\frac{M^2}{\lambda^2} - \left(\log\frac{s}{m_e^2}-1\right) \\
&\times\left(2\log\frac{4(\Delta E)^2}{\lambda^2} - 2\log\frac{s}{m_e^2}\right) \\
&-2\left(\log\frac{s}{M^2}-1\right)\left(\log\frac{4(\Delta E)^2}{\lambda^2} - \log\frac{s}{M^2}\right) \\
&\left.-\log^2\frac{s}{m_e^2} - \log^2\frac{s}{M^2}\right] \\
&\left.+\frac{e^2}{2\pi^2}\log\frac{4(\Delta E)^2}{M^2}\log\frac{t}{u} + \frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f} Q_j^2 N_C^j \log\frac{M^2}{m_j^2}\right\} \\
&= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{Born}}_{-,\mathrm{L}} \\
&\times\left\{1 - \frac{e^2}{8\pi^2}\left[\frac{1+2c_{\mathrm{w}}^2}{2s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log^2\frac{s}{M^2} - 7\frac{1+2c_{\mathrm{w}}^2}{4s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log\frac{s}{M^2}\right.\right. \\
&+\frac{2}{s_{\mathrm{w}}^2}\log\frac{s}{M^2}\left(\frac{(1-2c_{\mathrm{w}}^2)^2}{2c_{\mathrm{w}}^2}\log\frac{t}{u} + 2c_{\mathrm{w}}^2\log\frac{-t}{s}\right)
\end{aligned}
$$

$$+\frac{3m_t^2}{2s_{\mathrm{w}}^2M^2}\log\frac{s}{m_t^2}-2\log^2\frac{s}{M^2}+3\log\frac{m_e^2}{M^2}$$
$$+4\log\frac{s}{4(\Delta E)^2}\left(\log\frac{s}{m_eM}+\log\frac{t}{u}-1\right)\Bigg]$$
$$\left.+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{M^2}{m_j^2}\right\}. \quad (41)$$

Adding (34) and (41) yields exactly the result in (26) from [28]. Analogously, we have for right handed electrons:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{+,\mathrm{L}}=\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{Born}}_{+,\mathrm{L}}$$
$$\times\left\{1-\left(\frac{g^2}{8\pi^2}T_\phi(T_\phi+1)+\frac{g'^2}{8\pi^2}\frac{Y_\phi^2}{4}\right)\right.$$
$$\times\left(\log^2\frac{s}{M^2}-4\log\frac{s}{M^2}\right)$$
$$-\left(\frac{g^2}{8\pi^2}T_{e^-_+}(T_{e^-_+}+1)+\frac{g'^2}{8\pi^2}\frac{Y^2_{e^-_+}}{4}\right)$$
$$\times\left(\log^2\frac{s}{M^2}-3\log\frac{s}{M^2}\right)-3\frac{g^2}{16\pi^2}\frac{m_t^2}{M^2}\log\frac{s}{m_t^2}$$
$$-\frac{g'^2}{4\pi^2}\log\frac{s}{M^2}\log\frac{t}{u}-\frac{e^2}{8\pi^2}\Bigg[\left(\log\frac{s}{m_e^2}-1\right)2\log\frac{m_e^2}{\lambda^2}$$
$$+\log^2\frac{s}{m_e^2}-3\log\frac{s}{m_e^2}-\log^2\frac{s}{M^2}+3\log\frac{s}{M^2}$$
$$+2\left(\log\frac{s}{M^2}-1\right)\log\frac{M^2}{\lambda^2}-\left(\log\frac{s}{m_e^2}-1\right)$$
$$\times\left(2\log\frac{4(\Delta E)^2}{\lambda^2}-2\log\frac{s}{m_e^2}\right)$$
$$-2\left(\log\frac{s}{M^2}-1\right)\left(\log\frac{4(\Delta E)^2}{\lambda^2}-\log\frac{s}{M^2}\right)$$
$$-\log^2\frac{s}{m_e^2}-\log^2\frac{s}{M^2}\Bigg]+\frac{e^2}{2\pi^2}\log\frac{4(\Delta E)^2}{M^2}\log\frac{t}{u}$$
$$\left.+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{M^2}{m_j^2}\right\}$$
$$=\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{Born}}_{+,\mathrm{L}}$$
$$\times\left\{1-\frac{e^2}{8\pi^2}\left[\frac{5-2c_{\mathrm{w}}^2}{4s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log^2\frac{s}{M^2}-\frac{4-c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log\frac{s}{M^2}\right.\right.$$
$$+\frac{2}{c_{\mathrm{w}}^2}\log\frac{s}{M^2}\left(1-2c_{\mathrm{w}}^2\right)\log\frac{t}{u}$$
$$-2\log^2\frac{s}{M^2}+3\log\frac{m_e^2}{M^2}+\frac{3m_t^2}{2s_{\mathrm{w}}^2M^2}\log\frac{s}{m_t^2}$$
$$+4\log\frac{s}{4(\Delta E)^2}\left(\log\frac{s}{m_eM}+\log\frac{t}{u}-1\right)\Bigg]$$
$$\left.+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{M^2}{m_j^2}\right\}. \quad (42)$$

Again we see that after adding (35) and (42) we obtain the result in (27) from [28]. Thus we have demonstrated that to subleading logarithmic accuracy our results from the infrared evolution equation method in conjunction with the Goldstone boson equivalence theorem are identical with existing one loop calculations with physical fields in the high energy limit.

In a similar fashion, at the one loop level all DL and SL results from the infrared evolution equation method agrees with the general one loop corrections obtained from the physical fields in [9].

At higher orders, only for the process of massless $e^+e^- \longrightarrow \overline{f}f$ production results are available in [11] to SL and in [12] even to SSL order.

The Born amplitude in the high energy limit for $f \neq e$ is given by

$$\mathcal{M}^{\mathrm{Born}}_{e^+_\alpha e^-_\alpha\longrightarrow\overline{f}_\beta f_\beta}=\frac{\mathrm{i}}{s}\left[g^2T^3_{e^-_\alpha}T^3_{f_\beta}+g'^2\frac{Y_{e^-_\alpha}Y_{f_\beta}}{4}\right]$$
$$\times\langle e^+,\alpha|\gamma_\mu|e^-,\alpha\rangle\langle\overline{f},\beta|\gamma^\mu|f,\beta\rangle. \quad (43)$$

In the high energy regime we have at one loop order for the angular corrections relative to the Born amplitude:

$$\sum_{B,W^a}\delta^\theta_{e^+_\alpha e^-_\alpha\longrightarrow\overline{f}_\beta f_\beta}$$
$$=-\frac{g^2}{16\pi^2}\log\frac{s}{M^2}\left\{\left[\tan^2\theta_{\mathrm{w}}Y_{e^-_\alpha}Y_{f_\beta}+4T^3_{e^-_\alpha}T^3_{f_\beta}\right]\log\frac{t}{u}\right.$$
$$+\frac{\delta_{\alpha,-}\delta_{\beta,-}}{\tan^2\theta_{\mathrm{w}}Y_{e^-_\alpha}Y_{f_\beta}/4+T^3_{e^-_\alpha}T^3_{f_\beta}}$$
$$\left.\times\left(\delta_{d,f}\log\frac{-t}{s}-\delta_{u,f}\log\frac{-u}{s}\right)\right\}, \quad (44)$$

where the last line only contributes for left handed ($-$) fermions and the d, u symbols denote the corresponding isospin quantum number of f. In addition we have the soft angular contributions which are given by

$$\sum_{k,l}w^\theta_{kl}(s,\lambda)=-\frac{e^2}{4\pi^2}Q_eQ_f\log\frac{s}{\lambda^2}\log\frac{t}{u}, \quad (45)$$

and the corresponding matching term with $\lambda \rightarrow M$. Subtracting (45, $\lambda \rightarrow M$) from (44) is in agreement with (50) of [11].

Also the higher order SL terms from (17) are in agreement with the results of [11] (up to Yukawa enhanced terms which are neglected in a massless theory). The importance of these and their phenomenology is discussed in [11,12,2].

5 Conclusions

In this paper we have put forth the arguments allowing for a consistent SL resummation of logarithmically enhanced terms in the SM. Our strategy uses as an input only the knowledge of analogous terms in unbroken massless non-Abelian theories and in QED (including mass terms). While DL and universal SL terms have already been treated in earlier publications all the arguments given here for the angular terms can also be understood as a general justification for the matching approach, since the constraints stemming from the real soft QED regime apply for all terms. Here we have focussed on the non-universal process dependent angular corrections originating from the exchange of the electroweak gauge bosons.

We have argued that spontaneous symmetry breaking has effectively no bearing on these particular terms. We have used the infrared finiteness of semi-inclusive cross sections to argue that the mass gap of the electroweak gauge bosons does not lead to new effects at higher orders since the soft real QED corrections fix all possible virtual terms containing $\log \lambda$. Similarly, all fermion mass singularities are fixed by the KLN theorem and in particular, are due only to photon exchange assuming $m_i \leq M$. This is also true for novel CKM mixing effects in the massive quark sector.

From direct inspection of the Feynman rules we have concluded that no Yukawa dependent angular terms arise which are not mass suppressed. On the other hand, the scalar sector yields angular terms at higher orders in analogy to a scalar non-Abelian theory after the application of the equivalence theorem.

The soft (CKM extended) isospin correlations are included in matrix form. This is meant to apply to those external lines for which an isospin can be assigned. For the photon and the Z-boson, the notation applies to the symmetric basis amplitudes only. The result can be formulated in exponentiated operator form in the (CKM extended) isospin space.

We have shown how to apply our results at the one loop level and find agreement with all known results. In addition, we have derived the two loop factorization leading to an exponentiated result for right handed massive fermions, demonstrating that fermion masses and the gauge boson mass gap does not spoil the higher order resummation in the SM. Also in the general case at higher orders we agree with the known corrections for massless four fermion processes.

In summary, we have shown that a full SL description for arbitrary electroweak processes is possible based on the concept of the infrared evolution equation method. The accompanying soft QED corrections can all be accommodated for through the appropriate matching conditions at the weak scale.

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